# Dynamic lot sizing with product returns

Ruud H. Teunter*

Department of Management Science, Lancaster University Management School,

Lancaster, LA1 4YX, UK,

tel/fax: +44-1524-594326/844885, e-mail: r.teunter@lancaster.ac.uk

Z. Pelin Bayındır

Department of Industrial and Systems Engineering, University of Florida,

303 Weil Hall, PO Box 116595, Gainesville, FL 32611-6595, USA,

tel(ext.)/fax: +1-352-3921464(2053)/3537, e-mail: bayindir@ise.ufl.edu

Wilco van den Heuvel

Econometric Institute and Erasmus Research Institute of Management,

Erasmus University Rotterdam,

PO Box 1738, Burg. Oudlaan 50, 3000 DR Rotterdam, The Netherlands,

tel/fax: +31-10-4081321/9162, e-mail: wvandenheuvel@few.eur.nl

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**Abstract**

We address the dynamic lot sizing problem for systems with product returns. The demand and return amounts are deterministic over the finite planning horizon. Demands can be satisfied by manufactured/procured new items, but also by remanufactured returned items. The objective is to determine those lot sizes for manufacturing and remanufacturing that

*corresponding author

minimize the total cost composed of holding cost for returns and serviceable products and set-ups costs. Two different set-up cost schemes are considered; there is either a joint set-up cost for manufacturing and remanufacturing (single production line) or separate set-up costs (dedicated production lines). For the joint set-up cost case, we present an exact, polynomial time dynamic programming algorithm. For both cases, we propose a number of heuristics and test them in an extensive numerical study.

Keywords: inventory management, lot sizing, reverse logistics

# 1 Introduction

Dynamic lot sizing, i.e., planning manufacturing/production orders over a number of future periods in which demand is dynamic and deterministic, is one of the most extensively researched topics in production and inventory control. See Silver, Pyke, and Peterson [15] for a general overview, and Brahimi et al. [2] for a recent and extensive review of single item models. However, the literature on dynamic lot sizing with product returns, where remanufacturing of those returns is an alternative for manufacturing, is very scarce.

Remanufacturing can be defined as the recovery of returned products, often involving disassembly, cleaning, testing, part replacement/repair, and reassembly operations, after which they are as-good-as-new. The latter term distinguishes remanufacturing from other recovery types such as material and energy recycling. See Thierry *et al.* [20] for a comparison of recovery types.

Environmental legislation, societal pressure, and economic opportunities have motivated many firms to get involved with product remanufacturing, especially over the past 10 years. Products that are nowadays remanufactured include machine tools, medical instruments, copiers, automobile engines, computers, aviation equipment, telephone equipment, and tires (see Ferrer [4] [5], Kandebo [10], Lund [12], Sivinski and Meegan [16], Sprow [17], and Thierry *et al.* [20]).

The scientific literature on remanufacturing and product recovery in general has also been growing at an increasing rate over the past two decades. Overviews are provided by Fleischmann *et al.* [6], Guide *et al.* [8], and Gupta and Gungor [9]. For a discussion of more recent results we refer to Dekker et al. [3]. However, there are just a few papers on lot sizing for the systems with remanufacturing option.

Richter and Sombrutzki [13] and Richter and Weber [14] study special cases of the problem. In the first model, it is assumed that the number of returns is sufficient for satisfying all demands without delay, and therefore manufacturing is not considered. The second model does consider the manufacturing option. However, results are only derived for the special case that the number of returns in the first period is at least as large as the total demand over the planning horizon. So in fact, the manufacturing option is not needed in the second model either, but may be used for economic reasons if holding returns is very costly.

Golany et al. [7] study the problem without restrictive assumptions on the number of returns. They show that the lot sizing problem with remanufacturing can be formulated as a network flow problem. Using this formulation, they prove that the problem is NP-hard for general concave costs. For the special case with linear costs and hence zero set-up costs, they provide a polynomial-time algorithm.

Beltran and Krass [1] study the lot sizing problem with returns that can be directly reused, i.e., for which no remanufacturing is needed. They show that it suffices to consider solutions that satisfy the "zero-inventory property", and use this property to develop a dynamic programming (DP) algorithm with cubic time-complexity that determines the optimal manufacturing and disposal decisions for the case of concave cost functions. If procurement cost and disposal cost are non-decreasing over time, then the problem can be solved in quadratic time.

In this paper we study the lot sizing problem with remanufacturing of returns, without restrictions on the number of returns and with set-up costs included. Two different set-up cost schemes are considered. In the first model variant, there is a joint set-up cost for manufacturing and remanufacturing, which is suitable if manufacturing and remanufacturing operations are performed on the same production line using the same production resources. In the second model variant, there are separate set-up costs for manufacturing and remanufacturing, in line with situations where there are separate production lines. Note that all four above discussed papers assumed separate cost functions for manufacturing and remanufacturing, but none proposed algorithms without return restrictions and with set-up costs included.

For the problem with a joint set-up cost, we will show that the zero-inventory property and a "remanufacture-first" property hold. Using those properties, we derive an exact DP algorithm and prove that its time-complexity is polynomial in the length of the planning horizon. The algorithm is a generalization of the famous Wagner-Whitin algorithm for the lot sizing problems without returns. We further propose generalizations of the Silver-Meal, Least Unit Cost, and Part Period Balancing heuristics, and test them in an extensive numerical experiment.

For the problem with separate set-up costs, we show that the above mentioned properties no longer hold. We again propose generalizations of the Silver-Meal, Least Unit Cost, and Part Period Balancing heuristics and test them in an extensive numerical experiment.

The remainder of the paper is organized as follows. In Section 2, the model is presented. The case with a joint set-up cost is treated in Section 3. The exact algorithm is presented, and the heuristics are described and tested. Section 4 deals with separate set-up costs. Heuristics are described and tested. We end with conclusions and offer directions for future research in Section 5.

## 2 Model

Table 1 lists the notations that will be used.

** PLACE TABLE 1 HERE **

We address lot sizing problems for systems with product returns. Demands and returns are known for all periods of the planning horizon. Demands can be satisfied by manufactured/procured new items and by remanufactured returned items. In Figure 1, a simple sketch of the system is depicted.

** PLACE FIGURE 1 HERE **

Note that there is no disposal option for returned products. Recent research by Van der Laan and Salomon [11] and Teunter and Vlachos [19] has shown that such an option will not lead to a considerable cost reduction, unless the return rate is unrealistically high (above 90%) and the demand rate is very small (less than 10 per year).

For readability, we will present the case that the manufacturing and the remanufacturing lead times are less than one period so that demands can be satisfied by (re)manufacturing in the same period. If the lead times are equal but more than one period, say $L$ periods, then the only adjustment needed is that demands are shifted backwards by $L$ periods, as is done in Materials Requirements Planning (MRP) systems.

The objective is to determine the production plan, i.e., the number of items manufactured and remanufactured at each period, such that the total cost over the planning horizon is minimized. The following cost items are included:

- fixed set-up cost of manufacturing and remanufacturing,
- inventory holding cost for returns and serviceables.

Note that by ignoring variable cost of manufacturing and remanufacturing we implicitly assume that these costs are time stationary. Since we exclude the disposal of returns option, all the returns arriving to the system are eventually remanufactured. In the long-run, lot sizing decisions therefore do not affect the total variable manufacturing and remanufacturing costs incurred.

It is assumed that the holding cost rate for serviceables is at least that for holding returns. Since remanufacturing adds value to an item, this is a practical assumption. For a detailed discussion on how to set holding cost rates in a system with remanufacturing of product returns, we refer interested readers to Teunter et al. [18].

The set-up costs are modelled in two different ways: either there is a joint set-up cost for manufacturing and remanufacturing, or there are separate costs. The first approach is suitable if manufacturing and remanufacturing operations are performed on the same production line using the same production resources, whereas the second is suitable for situations with dedicated production lines. We remark that in practice, there could also be mixed settings, e.g. a single production line with (partially) dedicated resources. Those situations could be modelled by having both a joint (major) set-up cost for starting production as well as separate (minor) set-up costs for manufacturing and remanufacturing. That more general modelling of set-up costs will not be considered here, but the heuristics that we propose can easily be adjusted for such a situation.

Without loss of generality, we can assume that the initial stocks of serviceables and returns are both zero and that there is a positive demand in the first period. To see this, consider the general problem with possibly non-zero stocks $I_0^r$ of returns and $I_0^s$ of serviceables at the end of period 0. It is obvious that the first set-up should be placed in the first period $f$ for which cumulative demand $\sum_{i=1}^{f} D_i$ is larger than $I_0^s$. The relevant lot sizing problem is therefore from that period $f$ with positive demand onwards, and starts with $I_0^s - \sum_{i=1}^{f-1} D_i < D_f$ serviceables and $I_0^r + \sum_{i=1}^{f-1} R_i$ returns in stock at the end of period $f-1$. This problem can easily be transformed to an equivalent problem with zero initial (at the end of period $f-1$) stocks by subtracting $I_0^s - \sum_{i=1}^{f-1} D_i$ from the demand in period $f$ and adding $I_0^r + \sum_{i=1}^{f-1} R_i$ to the return in period $f$. Therefore, any general lot sizing problem with non-zero initial stocks can be transformed to an equivalent problem with zero initial stocks and positive demand in the first period.

## 3 Joint set-up cost for manufacturing and remanufacturing

The lot sizing problem under the joint manufacturing and remanufacturing set-up cost can be modelled as a mixed integer linear programming problem (MILP) as follows:

$$\min \quad \sum_{t=1}^{T} \{K\delta_t + h^r I_t^r + h^s I_t^s\}$$

subject to

$$\begin{aligned}
I_{t-1}^r + R_t - x_t^r &= I_t^r \text{ for } t = 1, \ldots, T && (1)\\
I_{t-1}^s + x_t^r + x_t^m - D_t &= I_t^s \text{ for } t = 1, \ldots, T && (2)\\
x_t^r + x_t^m &\leq M_t\delta_t \text{ for } t = 1, \ldots, T && (3)
\end{aligned}$$

$$\delta_t \in \{0,1\},\ x_t^r,\ x_t^m,\ I_t^r,\ I_t^s \geq 0 \text{ for } t = 1, \ldots, T,$$

where $M_t = \sum_{i=t}^{T} D_i$ for $t = 1, \ldots, T$. Constraints (1) and (2) assure the inventory balance in return and serviceable stocks, respectively. Constraints (3) keep track of the set-ups; whenever a manufacturing or a remanufacturing lot is produced, a set-up is made and the production (the sum of the amount manufactured and remanufactured) in period $t$ will never exceed the total demand in periods $t, \ldots, T$.

Next, we will derive some optimality conditions that will provide the basis for an exact dynamic programming algorithm and a number of heuristics. The conditions are presented in the form of two lemmas.

The first lemma states that for any optimal solution, the stock at the beginning of any period with a set-up is zero. This lemma is a generalization of the well-known zero-inventory property for the original lot sizing problem without returns.

**Lemma 1** *Any optimal solution satisfies the zero-serviceable-inventory-property: for any period with a set-up it holds that the stock of serviceables at the beginning of the period is zero, i.e.,* $I^s_{t-1}\delta_t = 0$ *for* $t = 1, 2, \ldots, T$.

**Proof** Consider any solution $\pi$. Since the initial stock (at the end of period 0) of serviceables is zero, the property obviously holds for the first period (in which there is a set-up since demand is positive). Now consider any other period $t \geq 2$ with a set-up under solution $\pi$, and let $t'$ denote the preceding period with a set-up. So, $t'$ and $t$ are successive set-up periods under solution $\pi$. We shall complete the proof by showing that if the stock of serviceables is positive at the beginning of period $t$, then an alternative feasible solution $\pi'$ with lower cost can be constructed. We consider two cases. Case 1: $\pi$ only remanufactures in period $t'$. Then $\pi'$ remanufactures one less item in $t'$ and one more in $t$. Case 2: $\pi$ manufactures in period $t'$. Then $\pi'$ manufactures one less item in $t'$ and one more in $t$. For both cases, it is clear that $\pi'$ is feasible, and that $\pi$ and $\pi'$ have the same number of set-ups and therefore the same set-up cost. However, the difference in holding cost for returns and serviceables under solutions $\pi$ and $\pi'$ is $(h^s - h^r)(t - t') > 0$ for Case 1 and $h^s(t - t') > 0$ for Case 2. ■

We remark that Lemma 1 can also be used to solve the MILP more efficiently by adding the constraints $I^s_{t-1} \leq (1 - \delta_t) \sum_{i=t}^{T} D_i$ for $t = 1, 2, \ldots, T$.

The second lemma shows that priority is given to remanufacturing option, i.e. that any optimal solution only manufactures in a certain period if the initial stock of returns at the beginning of that period is insufficient for remanufacturing the entire lot.

**Lemma 2** *Any optimal solution satisfies the following property: in every period where items are manufactured, the stock of returns at the end of that period is zero, i.e.,* $I_t^r x_t^m = 0$ *for* $t = 1, 2, \ldots, T$.

**Proof** Consider any solution $\pi$ that does not satisfy the property. Then there must be some period $t$, $1 \leq t \leq T$, with manufacturing and with a positive stock of returns at the end. An alternative solution $\pi'$ is to manufacture one less item and remanufacture one more item in period $t$, and to manufacture one more item and remanufacture one less item during the first period after $t$ in which $\pi$ remanufactures (if there is remanufacturing after period $t$). Clearly, solution $\pi'$ is feasible. Policies $\pi$ and $\pi'$ have the same number of set-ups and therefore the same set-up costs. Furthermore, it is clear that they have the same holding cost for serviceables. However, solution $\pi'$ has a lower holding cost for returns. Therefore, solution $\pi$ cannot be optimal. ■

### 3.1 Exact dynamic programming algorithm

The dynamic programming algorithm that is proposed in this section is a generalized version of the one proposed by Wagner and Whitin [22] for solving the dynamic lot sizing problem without returns. See Table 1 for notations. Note that, for ease of presentation, some of the notations are also used for period 0.

The algorithm starts by considering period 0 only. Clearly, the stock of returns at the end of that period is 0. So, $S_0 = \{0\}$, $f_0(0) = 0$, and $f_0 = 0$. The algorithm then recursively solves the lot sizing problem until period $k = 1, 2, \ldots, T$ by deriving $S_{l,k}$ (for all $l = 1, \ldots, k$), $S_k$, $f_{l,k}(n)$ (for all $l = 1, \ldots, k$ and $n \in S_{l,k}$), $f_k(n)$ (for all $n \in S_k$), and $f_k$. The recursive equations are derived below.

Consider the problem until period $k$. If the last set-up is in period $l$, $l = 1, 2, \ldots, k$, then all the returns in periods $l+1$ until $k$, i.e. $\sum_{i=l+1}^{k} R_i$, will be in stock at the end of period $k$. Moreover, if there are returns left in stock at the end of period $l$, then those items will also be in stock at the end of period $k$. From Lemma 2 it follows that there can only be returns left in stock at the end of period $l$ if the stock at the end of period $l-1$ plus the returns in period $l$ is larger than the size of the order in period $l$. Using Lemma 1 it follows that the size of the order in period $l$ is $\sum_{i=l}^{k} D_i$. We therefore get

$$S_{l,k} = \bigcup_{j \in S_{l-1}} \left\{ \left( j + R_l - \sum_{i=l}^{k} D_i \right)^+ + \sum_{i=l+1}^{k} R_i \right\}. \tag{4}$$

Clearly, we further have

$$S_k = \bigcup_{l=1}^{k} S_{l,k}. \tag{5}$$

Next, we derive the recursive expression for $f_{l,k}(n)$. As explained above, if the last set-up is in period $l$, then the stock of returns at the end of period $k$ is equal to $\sum_{i=l+1}^{k} R_i$ plus any stock that may be left at the end of period $l$. Two cases are distinguished.

- *There is no stock left at the end of period $l$.* Since remanufacturing is preferred to manufacturing (Lemma 2), this case occurs if the stock at the end of period $l-1$ plus the returns $R_l$ in period $l$ is at most the order size $\sum_{i=l}^{k} D_i$ in period $l$. The associated holding cost for returns in periods $l, \ldots, k$ is $h^r \sum_{i=l+1}^{k}(k+1-i)R_i$, since the returns in period $i$, $i = l+1, \ldots, k$, incur a holding cost at the end of periods $i, \ldots, k$. The associated holding cost for serviceables in periods $l, \ldots, k$ is $h^s \sum\limits_{i=l+1}^{k} (i-l)D_i$, since the items that satisfy the demands in period $i$, $i = l+1, \ldots, k$, incur a holding cost at the end of periods $l, l+1, \ldots, k-1$.

- *Some stock is left at the end of period $l$.* Then the stock at the end of period $k$ is more than $\sum_{i=l+1}^{k} R_i$. To attain stock level $n$ at the end of period $k$, there should be $n - \sum_{i=l+1}^{k} R_i$ left at the end of period $l$ and hence (since Lemma 2 implies that there is no manufacturing) $n - \sum_{i=l+1}^{k} R_i - R_l + \sum_{i=l}^{k} D_i = n + \sum_{i=l}^{k}(D_i - R_i)$ left at the end of period $l-1$. The associated holding cost for returns in periods $l, \ldots, k$ is therefore $h^r \left( \left( n - \sum_{i=l+1}^{k} R_i \right)(k-l+1) + \sum_{i=l+1}^{k}(k+1-i)R_i \right) = h^r \left( n(k-l+1) - \sum_{i=l+1}^{k}(i-l)R_i \right)$. The associated holding cost for serviceables in periods $l, \ldots, k$ is the same as for the first case.

So, we get

$$f_{l,k}(n) = \begin{cases} \left( \min\limits_{j \in S_{l-1} | j + R_l \leq \sum\limits_{i=l}^{k} D_i} f_{l-1}(j) \right) + K + h^s \sum\limits_{i=l+1}^{k} (i-l) D_i \\ \qquad + h^r \sum\limits_{i=l+1}^{k} (k+1-i) R_i (i-l) D_i \quad \text{for} \quad n = \sum\limits_{i=l+1}^{k} R_i \\ \\ f_{l-1}(n + \sum\limits_{i=l}^{k} (D_i - R_i)) + K + h^s \sum\limits_{i=l+1}^{k} (i-l) D_i \\ \qquad + h^r \left( n(k-l+1) - \sum\limits_{i=l+1}^{k} (i-l) R_i \right) \quad \text{for} \quad n \in S_{l,k} \setminus \left\{ \sum\limits_{i=l+1}^{k} R_i \right\} \end{cases} \tag{6}$$

Clearly, we further have

$$f_k(n) = \min_{l \in \{1,\ldots,k\} | n \in S_{l,k}} f_{l,k}(n) \text{ for } n \in S_k \tag{7}$$

and

$$f_k = \min_{n \in S_k} f_k(n) \text{ for } k = 1, \ldots, T. \tag{8}$$

Using the above results, we formulate the algorithm in the frame below. Step 1 initializes the values for period 0 and then sets the current period $k$ to 1. In the recursive Step 2, the optimal lot sizes until period $k$ is determined. When $k = T$ is reached, the algorithm goes to the final Step 3, where the optimal lot sizes are determined 'backwards'. This backward determination of the optimal lot sizes is similar to that for the original lot sizing problem without product returns. We do not describe it mathematically, since that is straightforward and would require additional notations.

**Dynamic Programming algorithm for joint set-up cost**

**Step 1:** $S_0 := \{0\}$, $f_0(0) = 0$, $f_0 = f_0(0) = 0$. $k := 1$.

**Step 2:** For all $l = 1, \ldots, k$, determine $S_{l,k}$ using (4). Determine $S_k$ using (5).
For all $l = 1, \ldots, k$ and all $n \in S_{l,k}$, determine $f_{l,k}(n)$ using (6).
For all $n \in S_k$, determine $f_k(n)$ using (7). Determine $f_k$ using (8).

**Step 3:** If $k = T$ then go to Step 4.
else $k := k + 1$, and go to Step 2.

**Step 4:** The minimal cost until period $t$, $t = 1, \ldots, T$ is $f_T$.
The corresponding lot sizes can be determined backwards.

Below we will prove that the algorithm runs in polynomial time. We start by stating two lemmas on the cardinality of sets $S_k$ and $S_{l,k}$.

**Lemma 3** *For $1 \leq k \leq T$ we have that $|S_k| = O(k^2)$.*

**Proof** Consider some solution up to period $k$ satisfying Lemmas 1 and 2. By definition, the set $S_k$ contains all possible ending inventories of returns $I_k^r$ at the end of period $k$, possibly including zero. Now assume that we have some positive ending inventory, i.e., $I_k^r > 0$. Furthermore, let $q$, $1 \leq q \leq k$, be the last period before $k$ satisfying $I_{q-1}^r = 0$. So, $I_t^r > 0$ for all $t = q, \ldots, k$. Such a period exists because $I_0^r = 0$. We consider the following two cases.

- There is no production in periods $q, \ldots, k$. This implies that all returns in those periods accumulate, so that

$$I_k^r = \sum_{t=q}^{k} R_t.$$

  Note that there is positive demand and therefore production in period 1. Hence, $q \geq 2$ and this case gives $k - 1$ possible values of $I_k^r$.

- Let $p$, $q \leq p \leq k$, be the first period after $q$ with non-zero production. Because $I_t^r > 0$ for $t = q, \ldots, k$, there is no manufacturing in periods $p, \ldots, k$. Otherwise Lemma 2 would be violated. This implies that all demands in periods $p, \ldots, k$ must be satisfied by the returns in periods $q, \ldots, k$ (if possible), so that

  $$I_k^r = \sum_{t=q}^{k} R_t - \sum_{t=p}^{k} D_t.$$

  Note that $q = 1$ implies that $p = 1$. Furthermore, because $q \leq p \leq k$, for $q \geq 2$ this case gives $\frac{1}{2}k(k-1) + 1$ possible values of $I_k^r$.

So in total, we have $1 + (k-1) + (\frac{1}{2}k(k-1)+1) = \frac{1}{2}k(k+1) + 1 = O(k^2)$ possible values of $I_k^r$, which implies that $|S_k| = O(k^2)$. ■

**Lemma 4** *For* $1 \leq k \leq T$ *we have that* $|S_{l,k}| = O((l-1)^2)$.

**Proof** This follows directly from (4) and Lemma 3. ■

**Theorem 1** *The DP algorithm runs in* $O(T^4)$ *time.*

**Proof** We will show that the time it takes to calculate all values of (6), (7), and (8) is at most $O(T^4)$ for each of the equations.

- *Equation (6)* Consider any fixed periods $k$ and $l$ for which $1 \leq l \leq k \leq T$. It takes $O((l-1)^2)$ time to compute $f_{l,k}(n)$ for $n = \sum_{i=l+1}^{k} R_i$ because $|S_{l-1}| = O((l-1)^2)$ (see Lemma 4). Furthermore, for a fixed $n \in S_{l,k} \setminus \left\{\sum_{i=l+1}^{k} R_i\right\}$ it takes constant time to calculate $f_{l,k}(n)$. So $f_{l,k}(n)$ can be computed in $O((l-1)^2)$ time for all $n \in S_{l,k} \setminus \left\{\sum_{i=l+1}^{k} R_i\right\}$, and hence $f_{l,k}(n)$ can be calculated in $O((l-1)^2)$ for all $n \in S_{l,k}$. Because there are $O(T^2)$ combinations of $k$ and $l$, the computation of $f_{l,k}(n)$ takes $O(T^4)$ time in total.

- *Equation (7)* For any fixed period $k$ $(1 \leq k \leq T)$ and fixed $n \in S_{l,k}$, the computation of $f_k(n)$ takes $O(k)$ time. Because $|S_{l,k}| = O((l-1)^2)$ (see Lemma 4) and $1 \leq k \leq T$, the computation of $f_k(n)$ can be performed in $O(T^4)$ time in total.

- *Equation (8)* Because $|S_k| = O(k^2)$ (see Lemma 3), it follows immediately that $f_k$ can be calculated in $O(k^2)$ time for any fixed period $k$ $(1 \leq k \leq T)$. So it takes $O(T^3)$ time to compute all values of $f_k$.

Therefore, the algorithm runs in $O(T^4) + O(T^4) + O(T^3) = O(T^4)$ time. ■

Although the optimal solution can be found in polynomial time, in practice one often uses heuristics to solve lot sizing problems. In general a heuristic approach is easier to understand and to implement (for example in an ERP package). Therefore, we consider some extensions of well-known lot sizing heuristics in the next section.

## 3.2 Heuristics

The most well-known heuristics for original lot sizing problems without returns are Silver Meal (SM), Least Unit Cost (LUC), and Part Period Balancing (PPB). See e.g. Silver, Pyke, and Peterson [15] for detailed descriptions. All three heuristics are myopic in the sense that they focus solely on the next order and ignore costs associated with future orders. Moreover, they only consider solutions that satisfy the zero-inventory property. The SM heuristic chooses the order that minimizes the cost per period. The LUC heuristic chooses the order that minimizes the cost per ordered unit. The PPB heuristic chooses the lot size that minimizes the difference between the set-up cost and the total holding cost. We propose modified versions of these three heuristics.

Since the zero-inventory property still holds for situations with returns (see Lemma 1) the restriction to solutions that satisfy this property remains justified. Further, based on Lemma 2, it is logical to restrict to *remanufacture-first* solutions. The only modification for the original heuristics that is required concerns the calculation of the total cost associated with an order. The modified cost expression is derived below.

Let $C_{l,k}(m)$ denote the total cost in interval $[l, k]$ if the stock of returns (determined by the previous lot-sizing decisions) at the end of period $l-1$ is $m$, and an order is placed in period $l$ that is sufficient until period $k$, that is of size $\sum_{i=l}^{k} D_i$. Clearly, there are returns left in stock at the end of period $l$ if and only if the stock $m$ at the end of period $l-1$ plus the returns $R_l$ in period $l$ is larger than the order size $\sum_{i=l}^{k} D_i$. If items are left in stock at the end of period $l$, then they will remain in stock until the end of period $k$. So, the associated return holding cost is $h^r(k-l+1)\left(m + R_l - \sum_{i=l}^{k} D_i\right)^+$. Expressions for the return holding costs associated with returns after period $l$ and for serviceable holding costs can easily be derived using arguments similar to those that lead to (6). This gives

$$C_{l,k}(m) = K + h^r \left( (k-l+1) \left( m + R_l - \sum_{i=l}^{k} D_i \right)^+ + \sum_{i=l+1}^{k} (k+1-i) R_i \right) + h^s \sum_{i=l+1}^{k} (i-l) D_i. \quad (9)$$

All three heuristics use this modified cost expression, and are otherwise unchanged from the original version.

### 3.3 Numerical experiment

Four different types of demand and return patterns are considered: stationary, linearly increasing, linearly decreasing, and seasonal. The mean return rate is set to either 30%, 50%, or 70% of mean demand. The total number of demand and return patterns considered are 10 and 18, respectively. For each pattern, four series of realizations are generated, so that the total number of demand and return series are 40 and 72, respectively.

The serviceable holding cost per period is normalized at 1. The remanufacturing holding cost is relatively small (0.2), moderate (0.5), or large (0.8). For the joint/separate set-up costs, 3 values are considered. We remark that based on some preliminary investigations, these cost values are chosen such that during the planning horizon, which is fixed at 12 periods, the number of periods with a set-up for the optimal solution varies between 2 and 6.

For details on the demand and return patterns, and on the cost parameter values, we refer to Table 2.

** PLACE TABLE 2 HERE **

A full factorial design is applied, so that the total number of examples is $40 \times 72 \times 3 \times 3 = 25,920$. We measure the performance of a heuristic by the percentage increase in the total cost compared to an optimal solution, which we refer to as the "error".

A first important result is that the average error over all examples for PPB (24.8%) is much larger than that for SM (3.0%) and LUC (4.2%). Apparently, balancing ordering and holding costs does not lead to a near-optimal solution. Indeed, further analysis of all optimal solutions revealed that, on average, the division of total cost into holding and set-up is not 50-50 but roughly 40-60. The poor performance of PPB can be further explained by the combined effect of fluctuations in returns and demands. Silver, Pyke, and Peterson [15] report that for traditional systems without returns, the performance of PPB is negatively effected by increased fluctuations.

Next, we perform a sensitivity analysis to determine the effects of the demand pattern, return pattern, return rate, and cost parameters on the performance of the heuristics. The results are summarized in Table 3. Our discussion of the results will therefore concentrate on the SM and LUC heuristics, because of their overall superior performance compared to PPB.

** PLACE TABLE 3 HERE **

It appears that under most demand and cost settings, SM performs slightly better then LUC. SM performs significantly better for seasonal demand, especially if fluctuations are large. The performances of both SM and LUC are quite robust with respect to the demand and return patterns. As expected, increased demand fluctuations generally (though not for all patterns) deteriorate the performances of SM and LUC. An increase in the return rate or in the unit holding cost of returns also affect the performances negatively. However, both heuristics have an average error of less than 5% under all scenarios for the return rate and for the return holding cost.

Cost values have a larger influence on the performances. It especially appears that the larger the set-up cost, the poorer the performance. However, it is important to realize that the poorer performance for larger set-up costs is (mainly) caused by the larger order sizes and hence smaller numbers of orders. Due to the myopic shortsighted nature of the heuristics, the last order in a heuristic solution is often much too small. For $K = 2000$ there are typically just 2 orders over the planning horizon, compared to 6 on average for $K = 200$, and therefore the relative effect on the cost of a too small last order is larger. In other words, the poorer performance for larger set-up costs is due to the relatively smaller planning horizon (in terms of number of orders placed) and does not result from unsuitability of the proposed heuristics for large $K$.

## 4 Separate set-up costs for manufacturing and remanufacturing

The MILP formulation is similar to that for the joint set-up cost case except for the separation of the set-up cost.

$$\min \quad \sum_{t=1}^{T} \{K^r \delta_t^r + K^m \delta_t^m + h^r I_t^r + h^s I_t^s\}$$

subject to

$$I_{t-1}^r + R_t - x_t^r = I_t^r \text{ for } t = 1, \ldots, T \tag{10}$$

$$I_{t-1}^r + x_t^r + x_t^m - D_t = I_t^s \text{ for } t = 1, \ldots, T \tag{11}$$

$$x_t^r \leq M_t \delta_t^r \text{ for } t = 1, \ldots, T \tag{12}$$

$$x_t^m \leq M_t \delta_t^m \text{ for } t = 1, \ldots, T \tag{13}$$

$$\delta_t^r,\ \delta_t^m \in \{0,1\},\ x_t^r,\ x_t^m,\ I_t^r,\ I_t^s \geq 0 \text{ and } M_t = \sum_{i=t}^{T} D_i \text{ for } t = 1, \ldots, T$$

Recall from Section 3 that for the case with a joint set-up cost, optimal solution for the problem satisfies zero-inventory and remanufacture-first properties. These properties enabled us to construct an exact algorithm of polynomial time-complexity. The following simple example shows that the properties no longer hold when there are separate set-up costs. Let $K^r = 10$, $K^m = 10$, $h^r = 1$, $h^s = 2$, $T = 2$, $D_1 = 2$, $D_2 = 100$, $R_1 = 1$, and $R_2 = 98$. It is easy to check that the optimal solution is to manufacture 3 products in period 1 and remanufacture 99 in period 2, with corresponding total cost 23. This solution satisfies neither of the two properties. Therefore, we are not able to develop a polynomial DP algorithm for the case with separate set-up costs.

In fact, *we conjecture that the problem with separate set-up costs is NP-hard*. Besides the fact that the zero-inventory property and the remanufacture-first property no longer hold, there is another result that strongly points in this direction. Van den Heuvel [21] shows that the problem becomes NP-hard when variable (re)manufacturing costs are included, even under the condition that the variable cost for manufacturing is larger than that for remanufacturing (which will typically hold if remanufacturing is motivated economically). It can easily be shown for the joint set-up cost problem with inclusion of variable costs under these conditions, that the two properties still hold, the DP algorithm can still be applied, and therefore the problem remains polynomially solvable. So, apparently, it is not the inclusion of variable costs but the separation of set-up costs which is responsible for the NP-hardness.

We will propose and test a number of heuristics. As for the joint set-up cost problem, these heuristics are generalized versions of the well-known Silver-Meal, Least Unit Cost, and Part Period Balancing heuristics for systems without remanufacturing. All three heuristics simultaneously determine the manufacturing and the remanufacturing order sizes. We also tested heuristics that determine the order sizes sequentially, first for remanufacturing and then for manufacturing based on the remaining 'net demand'. However, it turned out that their performances were very poor compared to those of the simultaneous heuristics. Therefore, the sequential heuristics will not be presented.

### 4.1 Heuristics

The generalized versions of the well-known Silver-Meal, Least Unit Cost, and Part Period Balancing heuristics only consider solutions that satisfy the zero-inventory property for serviceables. Although, as discussed above, Lemma 1 does not hold in general for the separate set-up cost problem, we expect that for most realistic cases, a near-optimal solution that satisfies the zero-inventory property exists. The extensive numerical study in Section 4.2 will confirm that the cost increase of the best zero-inventory solution compared to the optimal solution is generally less than 2%.

The heuristics do consider solutions that not always remanufacture first. This is essential, since there can be situations with large set-up costs and few returns available, where it is clearly better to keep the returns in stock for now and manufacture only, rather than to remanufacture as well as manufacture. Therefore, besides *remanufacture-first* orders, the heuristics consider *manufacture-only* orders. Next, we will derive cost expressions for both types of orders.

The expression for the cost associated with a remanufacture-first order is similar to that for the case with joint set-up costs. Expression (9) only needs to be modified by including the set-up costs if a (re)manufacturing order is placed. Let $I$(condition) denote the indicator function, which is one if the condition is satisfied and zero otherwise. Using similar arguments as those leading to (9), we get the following expression for the cost $CR_{l,k}(m)$ in interval $[l,k]$ if the returns stock at the end of period $l-1$ is $m$ and a remanufacture-first order is placed in $l$ that is sufficient until $k$.

$$\begin{aligned} CR_{l,k}(m) &= I(m+R_l>0)K^r + I(m+R_l<\sum_{i=l}^{k} D_i)K^m + h^s \sum_{i=l+1}^{k}(i-l)D_i \\ &\quad + h^r\left((k-l+1)\left(m+R_l-\sum_{i=l}^{k} D_i\right)^+ + \sum_{i=l+1}^{k}(k+1-i)R_i\right) \end{aligned} \tag{14}$$

If a manufacturing-only order is placed in period $l$ for interval $[l,k]$ and the returns stock at the end of period $l-1$ is $m$, then the returns stock at the end of period $i$, $i=l,\ldots,k$, is $m+\sum_{s=l}^{t} R_s$. The corresponding cost is therefore

$$CM_{l,k}(m) = K^m + h^s \sum_{i=l+1}^{k}(s-l)D_i + h^r\left((k-l+1)m + \sum_{i=l+1}^{k}(k+1-i)R_i\right) \tag{15}$$

Aside from the consideration of two order types and the corresponding modified cost expressions, the heuristics are identical to the original ones for systems with manufacturing only.

## 4.2 Numerical experiment

The same demand patterns, return patterns, and cost parameter values are considered as in the previous experiment, but of course the set-up costs for manufacturing and remanufacturing can now have different values. See Table 2. The total number of examples is $77,760$.

We start by justifying the focus on heuristics that only consider solutions satisfying the zero-inventory property. For 38.1% of the examples, there is in fact an optimal solution that satisfies this property. For 52.5% of the examples, the cost increase of the best zero-inventory solution (determined by solving the MILP model with additional restirctions in Cplex) compared to the optimal solution is less than one per cent. The average cost increase over all examples is less than two per cent.

The computational results can be found in Table 4.

** PLACE TABLE 4 HERE **

As for the case with a joint set-up cost, the first apparent result is that the PPB performs much worse than SM and LUC for all demand/return patterns and cost values. Furthermore, SM again performs slightly better than LUC under most scenarios. The average error over all examples is 8.3% for SM, 9.0% for LUC, and 19.8% for PPB.

Most of the sensitivity results with respect to demand/return patterns and costs are also similar to those for the joint set-up cost case. Recall that for the joint set-up cost case, it was observed that an increase in the set-up cost lead to a poorer performance of all heuristics. This was explained by reduced number of orders and hence the (relatively) stronger effect of a too small last order. For the separate set-up cost case, we see the same effect from an increase in the manufacturing set-up cost. However, an increase in the remanufacturing set-up cost has the opposite effect; the performances of all heuristics improve. A look at the optimal solutions provided the following explanation. In cases where remanufacturing set-up is more costly than a manufacturing set-up, the optimal solution often place no remanufacturing batches at all, and neither do the heuristic solutions. As is known from the literature, the heuristics perform well in pure manufacturing situations.

# 5 Conclusion

A general version of the lot sizing problem with remanufacturing of returns was analyzed. Two different models were considered with either joint or separate costs for manufacturing and remanufacturing set-ups. For both models, the lot sizing problem was formulated as a mixed integer program (MIP). The problem with a joint set-up cost turned out to be least complex. Based on the so-called *zero-inventory* and remanufacture-first properties, a polynomial DP algorithm was provided. For situations with separate set-up costs, these properties no longer hold and the problem is conjectured to be NP-hard.

In practise, heuristic procedures will often be preferred to the exact MIP formulations and even to the DP algorithm. We therefore presented modified versions of the well-known Silver-Meal (SM), Least Unit Cost (LUC), and Part Period Balancing (PPB) heuristics, both under a joint set-up cost and under separate set-up costs. An extensive numerical experiment revealed that for both models, SM performs slightly better than LUC and a lot better than PPB.

For the joint set-up cost model, the cost error (increase in total cost compared to the optimal solution) is just 3.0% on average. Moreover, the performance of SM is robust with respect to demand/return patterns and cost parameter values, as long as the set-up cost is not so large that the number of set-ups during the planning horizon reduces to 1 or 2. We therefore recommend the use of SM in practise for systems with a joint set-up cost.

For the separate set-up cost model, the performance of SM is also quite robust. However, the average cost error of 8.4% is considerable. Some improvement can be expected if SM is applied in a rolling horizon setting, which it is likely to be in practise, but the substantial cost error does provide an incentive to explore alternative heuristics. This is one direction for further research.

Other interesting research avenues are to prove the conjecture that the problem is NP-hard for separate set-up costs, and to modify and test heuristics for 'mixed' cases with a joint set-up cost as well as separate set-up costs, or with an even more general set-up cost function.

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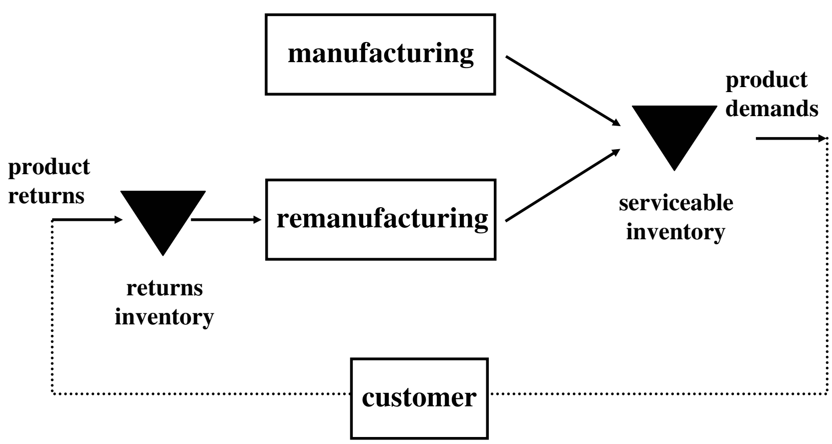


Figure 1: Inventory system with remanufacturing.

| | |
|---|---|
| **General** | |
| $T$ | Planning horizon |
| $t$ | Index for periods in the planning horizon, $t = 1, \ldots, T$ |
| $R_t$ | Number of returns received at the beginning of period $t$ |
| $D_t$ | Number of items demanded in period $t$ |
| $K$ | (joint) set-up cost |
| $K^r$ | (separate) Set-up cost for remanufacturing |
| $K^m$ | (separate) Set-up cost for manufacturing |
| $h^r$ | Unit holding cost for returns per period |
| $h^s$ | Unit holding cost of end-items (serviceables) per period |
| **Mixed integer linear programming (MILP) formulation** | |
| $x_t^r$ | Number of items remanufactured in period $t$ |
| $x_t^m$ | Number of items manufactured in period $t$ |
| $I_t^r$ | Inventory level of returns at the end of period $t$ |
| $I_t^s$ | Inventory level of serviceables at the end of period $t$ |
| $\delta_t$ | (joint) 0-1 indicator variable for remanufacturing set-up in period $t$ |
| $\delta_t^r$ | (separate) 0-1 indicator variable for remanufacturing set-up in period $t$ |
| $\delta_t^m$ | (separate) 0-1 indicator variable for manufacturing set-up in period $t$ |
| $M$ | Large integer |
| **Dynamic programming (DP) algorithm** | |
| $f_k$ | Minimum cost in periods $1, \ldots, k$ (if periods $k+1, \ldots, T$ are ignored) |
| $f_k(n)$ | Minimum cost in periods $1, \ldots, k$ (if periods $k+1, \ldots, T$ are ignored) with $n$ returns in stock at the end of period $k$ |
| $f_{l,k}(n)$ | Minimum cost in periods $1, \ldots, k$ if the last order is placed in periods $l$ with $n$ returns in stock at the end of period $k$ |
| $S_k$ | Set of possible returns stock levels at the end of period $k$ |
| $S_{l,k}$ | Set of possible returns stock levels at the end of period $k$ if the last order is placed in periods $l$ |
| **Heuristics** | |
| $C_{l,k}(m)$ | (joint) Cost in interval $[l, k]$ if the returns stock at the end of period $l-1$ is $m$ and an order is placed in period $l$ that is sufficient until period $k$ |
| $CR_{l,k}(m)$ | (separate) Cost in interval $[l, k]$ if the returns stock at the end of period $l-1$ is $m$ and a *remanufacture-first* order is placed in $l$ that is sufficient until $k$ |
| $CM_{l,k}(m)$ | (separate) Cost in interval $[l, k]$ if the returns stock at the end of period $l-1$ is $m$ and a *manufacture only* order is placed in $l$ that is sufficient until $k$ |
| $I$(condition) | Indicator function, which is one if the condition is satisfied and zero otherwise |

Table 1: Notations.

| Demand pattern | | | | | |
|---|---|---|---|---|---|
| $\mu$ | $\sigma$ | $\tau$ | $a$ | $c$ | $d$ |
| Stationary | | | | | |
| 100 | 10 | 0 | 0 | na | na |
| 100 | 20 | 0 | 0 | na | na |
| Positive trend | | | | | |
| 100 | 10 | 10 | 0 | na | na |
| 100 | 10 | 20 | 0 | na | na |
| Negative trend | | | | | |
| 210 | 10 | -10 | 0 | na | na |
| 320 | 10 | -20 | 0 | na | na |
| Seasonal (peak in middle) | | | | | |
| 100 | 10 | 0 | 20 | 12 | 1 |
| 100 | 10 | 0 | 40 | 12 | 1 |
| Seasonal (valley in middle) | | | | | |
| 100 | 10 | 0 | 20 | 12 | 3 |
| 100 | 10 | 0 | 40 | 12 | 3 |

| Return pattern | | | | | |
|---|---|---|---|---|---|
| $\mu$ | $\sigma$ | $\tau$ | $a$ | $c$ | $d$ |
| Stationary | | | | | |
| 30 | 3 | 0 | 0 | na | na |
| 30 | 6 | 0 | 0 | na | na |
| 50 | 5 | 0 | 0 | na | na |
| 50 | 10 | 0 | 0 | na | na |
| 70 | 7 | 0 | 0 | na | na |
| 70 | 14 | 0 | 0 | na | na |
| Positive trend | | | | | |
| 30 | 3 | 3 | 0 | na | na |
| 30 | 3 | 6 | 0 | na | na |
| 70 | 7 | 7 | 0 | na | na |
| 70 | 7 | 14 | 0 | na | na |
| Negative trend | | | | | |
| 63 | 3 | -3 | 0 | na | na |
| 96 | 3 | -6 | 0 | na | na |
| 147 | 7 | -7 | 0 | na | na |
| 224 | 7 | -14 | 0 | na | na |
| Seasonal (peak in middle) | | | | | |
| 30 | 3 | 0 | 6 | 12 | 1 |
| 30 | 3 | 0 | 12 | 12 | 1 |
| 70 | 7 | 0 | 14 | 12 | 1 |
| 70 | 7 | 0 | 28 | 12 | 1 |
| Seasonal (valley in middle) | | | | | |
| 30 | 3 | 0 | 6 | 12 | 3 |
| 30 | 3 | 0 | 12 | 12 | 3 |
| 70 | 7 | 0 | 14 | 12 | 3 |
| 70 | 7 | 0 | 28 | 12 | 3 |

Cost parameters

| Parameter | Values |
|---|---|
| $K$, $K^r$, $K^m$ | 200, 500, 2000 |
| $h^r$ | 0.2, 0.5, 0.8 |
| $h^s$ | 1 |

Table 2: Experimental Setting. The demand and return patterns are generated according to $D_t = \mu + \tau(t-1) + a \sin\left(\frac{2\pi t}{c} + d\frac{\pi}{2}\right) + \varepsilon_t$ for $t = 1, \ldots, T$, where $\mu$ is the starting level of the pattern, $\tau$ is the trend level, $a$ is the amplitude of the cycle, $c$ is the cycle length, $d$ determines the peak of the cycle and $\varepsilon_t$ $(t = 1, \ldots, T)$ are independently normally distributed random variables with standard deviation $\sigma$.

| | SM | LUC | PPB |
|---|---|---|---|
| **All instances** | 3.0 | 4.2 | 24.8 |
| **Demand pattern** | | | |
| Stationary | 3.3 | 3.5 | 32.5 |
| -Small variance | 2.8 | 2.4 | 31.9 |
| -Large variance | 3.8 | 4.6 | 33.0 |
| Positive trend | 2.0 | 2.4 | 10.2 |
| -Small trend | 1.3 | 3.4 | 12.2 |
| -Large trend | 2.7 | 1.4 | 8.1 |
| Negative trend | 3.2 | 3.0 | 15.9 |
| -Small trend | 3.6 | 2.5 | 15.8 |
| -Large trend | 2.8 | 3.5 | 15.9 |
| Seasonal | 3.3 | 6.1 | 32.7 |
| -Small amplitude | 3.3 | 4.6 | 30.5 |
| -Large amplitude | 3.3 | 7.6 | 34.8 |
| **Returns** | | | |
| Stationary | 2.7 | 3.8 | 16.4 |
| -Small variance | 2.7 | 3.8 | 16.3 |
| -Large variance | 2.7 | 3.8 | 16.6 |
| Positive trend | 3.9 | 5.1 | 34.8 |
| -Small trend | 3.5 | 4.5 | 29.4 |
| -Large trend | 4.3 | 5.8 | 40.2 |
| Negative trend | 3.3 | 4.8 | 41.0 |
| -Small trend | 3.5 | 4.8 | 34.4 |
| -Large trend | 3.1 | 4.8 | 47.7 |
| Seasonal | 2.7 | 3.8 | 17.9 |
| -Small amplitude | 2.6 | 3.7 | 17.8 |
| -Large amplitude | 2.7 | 3.8 | 18.0 |
| **Mean Return per period*** | | | |
| 0.3 | 2.1 | 3.0 | 5.0 |
| 0.5 | 2.8 | 4.0 | 13.7 |
| 0.7 | 3.3 | 4.4 | 30.6 |
| **Set-up cost** $K$ | | | |
| 200 | 1.1 | 1.5 | 8.5 |
| 500 | 1.9 | 2.4 | 16.7 |
| 2000 | 6.1 | 8.8 | 49.1 |
| **Returns holding cost** $h^r$ | | | |
| 0.2 | 2.0 | 2.9 | 26.2 |
| 0.5 | 3.2 | 4.6 | 24.8 |
| 0.8 | 3.8 | 5.1 | 23.3 |

* Only examples with stationary returns are considered.

Table 3: Sensitivity analysis on the performance (percentage cost error) of the SM, LUC, and PPB heuristics for the case with a joint set-up cost.

| | SM | LUC | PPB |
|---|---|---|---|
| **All instances** | 8.3 | 9.0 | 19.8 |
| **Demand** | | | |
| Stationary | 8.1 | 8.7 | 21.4 |
| -Small variance | 7.6 | 7.8 | 21.2 |
| -Large variance | 8.6 | 9.5 | 21.7 |
| Positive trend | 7.4 | 7.7 | 15.8 |
| -Small trend | 7.4 | 7.8 | 16.8 |
| -Large trend | 7.4 | 7.7 | 14.8 |
| Negative trend | 9.5 | 9.3 | 17.2 |
| -Small trend | 9.4 | 9.0 | 17.7 |
| -Large trend | 9.6 | 9.6 | 16.7 |
| Seasonal | 8.3 | 9.6 | 22.3 |
| -Small amplitude | 7.8 | 8.5 | 21.6 |
| -Large amplitude | 8.8 | 10.8 | 23.0 |
| **Returns** | | | |
| Stationary | 7.5 | 8.2 | 16.2 |
| -Small variance | 7.5 | 8.2 | 16.2 |
| -Large variance | 7.4 | 8.2 | 16.3 |
| Positive trend | 10.9 | 11.6 | 23.1 |
| -Small trend | 10.4 | 11.1 | 21.2 |
| -Large trend | 11.4 | 12.1 | 25.0 |
| Negative trend | 8.9 | 9.9 | 30.7 |
| -Small trend | 8.0 | 9.2 | 30.4 |
| -Large trend | 9.8 | 10.5 | 31.0 |
| Seasonal | 7.3 | 7.9 | 15.4 |
| -Small amplitude | 7.3 | 7.8 | 15.4 |
| -Large amplitude | 7.3 | 7.9 | 15.5 |
| **Mean Return per period*** | | | |
| 0.3 | 5.6 | 6.5 | 11.0 |
| 0.5 | 8.0 | 8.9 | 18.0 |
| 0.7 | 8.9 | 9.3 | 19.7 |
| **Manufacturing set-up cost** $K^m$ | | | |
| 200 | 6.2 | 7.6 | 17.0 |
| 500 | 6.9 | 7.2 | 18.0 |
| 2000 | 11.8 | 12.2 | 24.6 |
| **Remanufacturing set-up cost** $K^r$ | | | |
| 200 | 11.3 | 11.3 | 27.1 |
| 500 | 9.0 | 9.6 | 21.5 |
| 2000 | 4.7 | 6.1 | 10.8 |
| **Returns holding cost** $h^r$ | | | |
| 0.2 | 5.5 | 6.0 | 12.1 |
| 0.5 | 8.3 | 9.1 | 22.1 |
| 0.8 | 11.2 | 11.9 | 25.4 |

* Only examples with stationary returns are considered.

Table 4: Sensitivity analysis on the performance (percentage cost error) of the SM, LUC, and PPB heuristics for the case with separate set-up costs.